Guide to Preparing a Math Club for Contests

Including a Sample Syllabus and Example Problems

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INTRODUCTION

This document is intended as a helpful starting point for Math Club advisors/mentors/coaches in preparing students in 4^{th} – 8^{th} grades for math competitions (although this document is probably also helpful for non-competitive/enrichment Math Club approaches). This document can be one resource to add to your toolbox, but is not intended to be a comprehensive "how to" for running a Math Club.

This document provides some introductory discussion, a syllabus, and discussion of syllabus topics with example problems. In the introductory discussion, recommendations are given for several math contests (both in-person and "mail-in") and general considerations for contest preparation are touched upon. A syllabus is provided to give an idea of the type of activities and the associated timeframe when they could/should occur. Mathematical topic areas are discussed to give an introduction to the types of concepts to present to students. Example problems are provided for each topic area, including problem solutions. Finally, selected additional resources are itemized as a starting point for further exploration.

Using this Guide

Use the example syllabus and the topic area discussion/examples as a guide to the type of material to cover, the nominal timeframe for activities, and as starter problems to go over with students. It would seem most useful to first impart an understanding of the topic areas, using the example problems (or other material) to illustrate the concepts presented. Contact the Team Battelle Math Mentors/Coaches project director (see http://regionaloutreach.pnnl.gov/team-battelle/TBMath/MathClubMentors.htm) if you have questions or comments about this document or about Math Clubs in the Tri-Cities area.

Suggested Contests

Given the presumption of preparing students for math competitions, it is certainly important to know what competitions are available and how they work. Having targets for contest participation provides a framework for preparation activities in terms of practicing for specific test formats and types of problems. There are a number of competition opportunities (which vary by grade level), all with slightly different rules and format. Discussion here is limited to a few recommended contests, but information on other competitions can be found in the section below on List of Math Competitions, at the Team Battelle Math Mentors/Coaches project website (http://regionaloutreach.pnnl.gov/team-battelle/TBMath/MathClubMentors.htm#opportunities), and elsewhere on the Internet. Note that recommendations are the author's opinion; please make an evaluation for yourself.

There are several good mail-in math competitions that are similar in nature, but the Mathematical Olympiads for Elementary and Middle Schools (MOEMS) is recommended because it is both cost effective and the program does a good job of recognizing students. MOEMS consists of five rounds of tests, each with 5 questions. The test rounds are taken by students once per month from November to March. The Math Club advisor administers this individual test to the group of students, corrects the submitted answers, and then uploads the score information to the MOEMS website. Rounds are timed, but there is enough time that students shouldn't feel rushed. Two competition levels are offered, one for 4^{th}-6^{th} grades (Division E) and one for 6^{th}-8^{th} grades (Division M). Registration is for a team of up to 35 students and costs on the order of

$100. Each team receives certificates for all participants and a top scoring student trophy, plus patches and pins for students in the top 50%, 10%, and 2% nationwide. The top 10 individual student scores combined to make the team score, which is recognized with a plaque if it falls in the top 20% or 10% of all team scores. There are about 80,000 students across the nation who participate in Division E and another 20,000 in Division M. If you can only do one mail-in competition, then MOEMS is the one recommended.

The Mathematics Association of America (MAA) sponsors the AMC-8 contest, which is administered to middle school students at their school by the Math Club advisor. Answer sheets are collected and mailed off for scoring by MAA. This multiple choice contest is administered to individuals, but the school vies for national recognition using a team score comprised from the top 3 individual scores. AMC-8 is a good contest for participation by middle school students because it is a high caliber nation-wide contest. Good performance on AMC-8 can not only garner awards (pin) for a student, but can get the student noticed by programs looking for talented youth. Participation cost is $35 per school plus $12 per bundle of 10 tests. The AMC-8 program offers a "Math Club" package for purchase ($25 in recent years) that entails a book of information, activities, problems, and resource lists.

MATHCOUNTS is an excellent program with backing from the National Society of Professional Engineers and major engineering/technical companies (e.g. Raytheon, 3M, ConocoPhillips, etc.). MATHCOUNTS provides great resources (problems, activities) and has tangible perks for competitors (e.g., free lunch, participant freebies, state competition at Microsoft, etc.). Accordingly, the competition has more of a "this is serious business" flavor. The MATHCOUNTS competition is an in-person event for middle school students only and has successive local, state, and national levels of competition. Each competition includes four events that are done as individuals or as a team of four students. Cost for competing in MATHCOUNTS is on the order of $200 for a slate of 8 students (each school is limited to one official team of 4 and up to 6 individuals), but the club program and associated materials are free. Indeed the handbook and the silver-level challenges provide a good source of problems that have associated written solutions (not just answers).

The Math Is Cool (MIC) program for 4th to 12th grades was conceived of right here in Washington State (Spokane, actually) and is a great in-person contest. Math Is Cool consists of two individual events and four team events, including the ever-popular college bowl (quiz bowl) rounds. Squads of four students work together in the team events, with each school allowed multiple squads. Individual and school ("team") awards are given out. Competition starts at a regional level, with the higher-scoring teams moving on to state level competition. Cost is $40 per squad of 4, plus $10 per grade level. This competition is decidedly recommended because of the variety in the events and because students really have a good time.

Considerations When Preparing for Contests

The timing of when competitions occur will play into your strategy for preparation. Generally speaking, competitions for higher grade levels occur/start earlier in the school year. Thus, elementary level students typically have longer to prepare. The competition season really starts to get going in around November, which is when AMC-8 happens, when MOEMS starts, and when the regional level 7th and 8th grade Math Is Cool tournament takes place (state level in December). MATHCOUNTS chapter (regional) level tournaments happen in February, with the state level event in March. Math Is Cool regional competitions for 6th, 5th, and 4th grades occur in February, March, and April, respectively, with the state level event for those grades in May. The syllabus presented here will meet the needs of middle school students. As needed (i.e., for Elementary level Math Clubs), the timing can be adjusted to spread topics out, cover more types of problems, and/or intersperse other enrichment type activities (e.g., videos, math games).

Whether you select one or more of the recommended competitions, or you include other contests, you need to consider the nature of the competition when preparing students. All competitions cover the same types of topics (geometry, probability, etc.), but some may lean towards a certain style and there can be differing levels of rigor needed (e.g., fill in the blank vs. multiple choice). Practice for each competition is important so that students understand (or work out) strategies that can improve their effectiveness. For example, it eats up time if a squad of 4 students has to negotiate which student does what problem during an actual competition test. Similarly, you don't want the squad of students to be doing a Math Is Cool relay event for the first time at the competition. Also, working through problems helps the student get an idea of what to expect and exposes them to the breadth of problem types (and the associated tricks for solving the problem). The syllabus presented here includes times for practice with prior-year tests and review of the problems/solutions. Practice should involve not only the students calculating answers, but also a discussion of the solutions and related concepts. Actual tests from previous years and additional problems for practice can be found online or via other resources (e.g., other coaches/mentors).

It is to be expected that some students will find problems at their grade level difficult while other students need something even more challenging than what is presented. Problem difficulty for a competition will vary within a test as well as between levels of competition (e.g., regional versus state levels).

In the "real world" units of measure are a key component to the result of a calculation, but the use of units in contest answers varies. It is optional to provide units in Math Is Cool answers except for AM/PM on times. MATHCOUNTS always provides units labels next to answer blanks. Discuss the usefulness of paying attention to units when solving a problem and the contest-specific strategy for units in the answers.

Some competitions allow calculator use for all or a portion of the events. However, with few exceptions, contest problems are designed to be solvable without a calculator. I would encourage the Math Club advisor to have students work problems without calculators to improve their number manipulation skills and to better understand the pertinent solution techniques.

A Math Club coach need not be an expert at solving math problems, although having some skill is useful when going over solutions with students (or deriving a solution if one wasn't provided). However, one can often apply other resources (books, websites, parents, volunteers, etc.) to supplement the advisor/coach. The Team Battelle project director is a good source for materials/information and networking opportunities.

SYLLABUS

Two syllabi are presented here: one for elementary grades and one targeted at middle schools. It is assumed that the Math Club meets weekly, although a reduced or increased frequency may be helpful depending on what events are on the horizon. The syllabus lists a sequence of topics to cover in meetings, the approximate times of notable contests, and times when students should be practicing for a contest. The subsequent section discusses the topic areas and provides example problems.

Syllabus for Elementary Level Math Clubs

Month	Week #	Activity
September	1	Math Club kickoff meeting • Get student information (list of participants, home room, etc.) • Describe club activities, opportunities, exciting things to look forward to, etc. • Give a quiz so you'll have an idea of the knowledge/ability levels of students • Bring food and/or an icebreaker/activity
	2	Math Topics – vocabulary, problem solving strategies
	3	Math Topics – algebra
	4	Math Topics – number theory (factors, divisibility, etc.)
October	5	Math Topics – geometry
	6	Math Topics – combinatorics (combinations/permutations, sets)
	7	Math Topics – probability
	8	Math Topics – statistics and miscellaneous (logic, date/time, units, etc.
November	9	MOEMS Practice
	10	MOEMS #1
	11	Review MOEMS #1 solutions
	12	Math Topics – rate problems
December	13	Math Topics – real life/physics-based problems
	14	MOEMS #2
	15	Review MOEMS #2 solutions
	16	— Winter Break —
January	17	Introduction to Math Is Cool & CMS Warm-up formats
	18	MOEMS #3
	19	Review MOEMS #3 solutions
	20	Math League practice CMS Elementary Level Math Warm-up Tournament (Carmichael M.S.)
Februar y	21	Math League practice
	22	MOEMS #4
	23	Math League test
	24	5th MIC Practice
March	25	5th MIC Practice
	26	MOEMS #5
	27	4th/5th MIC Practice 5th Grade Math Is Cool (Carmichael M.S.)
	28	4th MIC Practice
April	29	— Spring Break —
	30	4th MIC Practice 4th Grade Math Is Cool (Carmichael M.S.)
	31	WAMO Practice
	32	4th/5th MIC Practice
May	33	4th/5th MIC Practice WAMO Competition (Richland)
	34	4th/5th MIC Practice MIC 4th/5th Masters (Moses Lake)
	35	End-of-Year Meeting (food, awards, season wrap-up)
	36	

Syllabus for Middle School Level Math Clubs

Month	Week #	Activity
September	1	Math Club kickoff meeting • Get list of students with grade level, current math class, contact info., food allergy info., etc. • Describe club activities, opportunities, exciting things to look forward to, etc. • Give a quiz so you'll have an idea of the knowledge/ability levels of students • Bring food and/or an icebreaker/activity
	2	Math Topics – vocabulary, algebra, problem solving strategies
	3	Math Topics – number theory (factors, divisibility, etc.)
	4	Math Topics – geometry
October	5	Math Topics – combinatorics (combinations/permutations, sets)
	6	Math Topics – probability 7th/8th MIC Practice
	7	Math Topics – statistics and miscellaneous 7th/8th MIC Practice CMS Middle School Math Warm-up Tournament (Carmichael M.S.)
	8	7th/8th MIC Practice
November	9	7th/8th MIC Regionals (Spokane)
	10	AMC-8 MOEMS #1
	11	7th/8th MIC Practice
	12	7th/8th MIC Practice
December	13	7th/8th MIC Masters (Moses Lake)
	14	MOEMS #2
	15	MATHCOUNTS Practice
	16	— Winter Break —
January	17	6th MIC Practice
	18	MOEMS #3 6th MIC Practice
	19	MATHCOUNTS School Test
	20	6th MIC Practice
February	21	6th MIC Practice
	22	MOEMS #4 6th MIC Regionals (Enterprise M.S.)
	23	Math League test MATHCOUNTS Chapter Level Competition
	24	MATHCOUNTS Silver Level Challenges
March	25	MATHCOUNTS Ultimate Math Challenge
	26	MOEMS #5
	27	MATHCOUNTS State Competition (Redmond)
	28	WAMO / Gauss / Purple Comet practice
April	29	— Spring Break —
	30	Purple Comet test
	31	6th MIC Practice
	32	6th MIC Practice
May	33	6th MIC Practice WAMO Competition (Richland)
	34	Gauss test MIC 6th Masters (Moses Lake)
	35	End-of-Year Meeting (food, awards, season wrap-up)
	36	

SYLLABUS TOPICS

The sections below describe major mathematical topic areas and the associated key concepts. Topic areas are not described formally (i.e., rigorous mathematical definitions), but are discussed in terms relative to the concepts important to students and the math contests in which they may participate. Examples (from prior-year Math Is Cool tests) are provided to illustrate typical problem types that are encountered. Check out complete Math Is Cool prior-year tests for an even broader range of problems to draw from. The example problems note the type of test (primarily differentiating quick answer versus longer problems) and the intended grade level, which serve as an indication of problem difficulty[1]. While this material is not a full-blown lesson plan, it should give a Math Club advisor/coach direction on the type of material to discuss. The approach presented in the syllabus involves first presenting the concepts (i.e., using the examples provided here as a starting point), followed by practice sessions with prior year tests. Math Club advisors could present material in a lecture format, but may want to consider the use of manipulatives, videos, games, and/or peer-to-peer teaching as alternate/complementary methods to elucidate concepts.

While the Math Is Cool prior-year tests provide an extensive set of material for practice, one drawback is that there are no associated solutions, meaning that the coach must know how to solve the problems (or get help from someone who does). Where available, prior year AMC-8, MOEMS, and MATHCOUNTS tests will usually have full solutions.

Problem Solving Strategies

While not a mathematical topic area or type of problem, this section discusses problem solving strategies in two senses: attacking unfamiliar problems and applying shortcuts.

Exposure to and discussion of a wide variety of math problems provides an opportunity for students to learn techniques and shortcuts, but it is useful to have a solid foundation in general problem solving techniques to provide context for dealing with unfamiliar problems and insight into all problems. Most information on mathematical problem solving can be traced back to the classic 1945 work by George Polya entitled “How To Solve It.” Polya discussed a four-step approach to problem solving, which is described below along with brief discussion of a suite of problem solving strategies. A number of books discuss and demonstrate approaches to solving math problems; selected titles are listed in the Problem Solving resources section of this document. Two websites with useful information on problem solving are:

- http://cte.uwaterloo.ca/teaching_resources/tips/teaching_problem_solving_skills.html and
- http://teachingtoday.glencoe.com/howtoarticles/promoting-problem-solving-skills-in-elementary-mathematics

[1] The “source” column contains shorthand notation describing the source and thus the nature of the question and the nominal difficulty. The shorthand notation of “N xxx yyyy, zz-p[.p]” indicates the grade level (N), the competition level (xxx → mic = regional, mas = state/masters), the year of the test (yyyy) as the calendar year in which the school year starts, the event (zz → i = individual, t = team test, mc = multiple choice, mm = mental math, cb = college bowl, pr = pressure round), and the problem number (p[.p]). Mental math and college bowl use the p.p format with person number or college bowl round, decimal point, problem number. For the individual test (and sporadically for other events), the higher the problem number, the greater the problem difficulty. Pressure round, mental math, and college bowl are all quick-answer type questions, versus the more involved problems likely to be encountered in team, multiple choice, or individual test events. This information allows you to find the tests (from the Math Is Cool website) and to quickly assess the nature of the problem difficulty.

There are typically four steps proposed as the overall approach to solving problems. First, one should ascertain what is known. The problem statement should be carefully read to identify important information (and discarding extraneous information) and the question that is being asked. Equally important is to assess what outside information (e.g., formulas, rules, properties, etc.) is known. The second step is to select a problem-solving strategy from the suite of "tools" available. Next, solve the problem by applying the selected strategy, realizing that one may find that a different strategy would be more helpful. Finally, assess the answer that was obtained to confirm that the answer makes sense and that it addresses the question that was asked.

There is a suite of problem-solving strategies that one can apply when working an unfamiliar problem. Some strategies, such as "Make a Model" and "Act it Out," are not well suited for the competition environment, but can find a place in the classroom or during practice sessions. Eleven strategies for problem solving are briefly described in the following list.

Compute/Use a Formula	Some problems merely require that you apply a formula or mathematical rules to simplify and compute the answer.
Make a Diagram or Plot	Sketch a diagram or plot to better visualize the problem. Or add to an existing diagram to help see what wasn't originally drawn. Be careful about eyeballing the size or spatial relationship of figures that are not drawn precisely or to scale.
Make a Table or List	Make a table or list to help get organized. For example, a table of the sums of two dice may be useful in solving a probability problem. Another example might be listing out permutations.
Guess, Check, and Revise	Make a reasonable (intelligent) guess and check it, revising the guess as needed.
Look for a Pattern	Look for patterns, which may have a visual component (as well as a numerical equivalent), then either calculate the next few steps or use an equation defining the pattern to get an answer.
Consider a Simpler Case	Consider a simpler case that helps you think through the numbers and what is happening. Sometimes the simpler case represents bounding conditions that can provide insight into the problem scenario.
Restate the Problem	Consider restating the problem in another way, perhaps from the converse perspective. For example, it may be easier to look at a probability problem from the perspective that event A does not occur instead of when event A does occur.
Eliminate	Eliminate possible solutions based on the information available and your knowledge (i.e., sometimes you can do a quick calculation to bound the answers or see that an answer is unrealistic, such as a probability of 120%).
Work Backward	Working backwards may provide an alternate perspective that makes problem solution easier. For example, in a problem describing where Jane spent her money (this much here, that percent there), working backwards could be an effective approach to determine how much money she started with.

Estimate	Use approximations to bound the answer or eliminate answers. For example, in a problem comparing the area of a circle to the area of a square, one could approximate π as 3.
Break into Parts	Break a larger or more complicated problem up into smaller, more manageable pieces that can be solved and subsequently combined.

Keep in mind that many mathematics problems have more than one method of solution, so multiple strategies may apply to any particular problem.

The above discussion describes the overall approach and strategies for solving problems, but there are other aspects to consider, particularly in the context of a timed contest, as problem-solving/test-taking strategies. These strategies pertain to working quickly while maintaining accuracy.

It is rather important to work towards eliminating "silly mistakes." Nothing is quite as frustrating as essentially getting the correct answer, but having it marked wrong because of a silly mistake (e.g., you forgot to divide by two because the problem asked about sharing with your brother). Richard Rusczyk (from the Art of Problem Solving) points out that two keys to minimizing stupid mistakes are developing good habits and organizing your work. Good habits include reading carefully and re-reading the problem statement, so that you avoid misreading the information or not answering the question that was asked. Organizing your work will help you keep track of what you are doing with the calculations and units. Writing higgledy-piggeldy wherever there is open space will result in misreading numbers or forgetting what you are doing.

When speed matters, mental calculations and writing in a shorthand notation can help. Both the Mental Math and College Bowl events of the Math Is Cool competition require the ability to think quickly. In Mental Math the student must do all calculations mentally, so it is important that students practice manipulating numbers in their head to become comfortable with that approach to visualizing and solving a problem. When proficient at performing operations (addition/subtraction/multiplication/division) mentally, the student may find that they are able to solve problems in other events (individual test, team test, etc.) more quickly. Sometimes, however it is most useful to write information down. Rather than writing out information (i.e., equations, figures) in precise or formal detail, students can use approximate diagrams and shorthand notation. Figures need only be sufficiently detailed to visualize the problem—for example, an oval will suffice for a circle and a square doesn't have to actually be square. Similarly, the student need not write out every step in solving an equation. Rather the manipulations (e.g., "divide both sides by …" or "subtract … from both sides") can be done mentally, and the result (perhaps of several manipulation steps) written down. Another aspect is to pick meaningful notation that is quick to write. When a problem involves Albert and Biff, the student could use "A" and "B" as the variables in an equation, instead of writing out the full names. And write notation in a quick manner—instead of writing a capital "E" (which takes 4 strokes), write a lowercase "e" (which takes 1 stroke). As an example, take the College Bowl question "Jamie is 16 years old and Jessica is 12 years old. In years, what will the sum of their ages be 3 years from now?" Given that this question is presented verbally, one might write the following:

$j = 16$
$e = 12$
$+3$

Here, the key information from the first sentence is written down because the actual question is unknown at that point. Then, with the question known from the second sentence, the student can mentally add 16 + 12 + 6 to get 34 without the need to write the full addition problem or the result. While approximate figures and shorthand notation will help with speed, be careful to write in a meaningfully way—it is unhelpful if you can't read what you wrote, you get lost in cryptic annotations, or the figures are insufficient to grasp the problem.

It is common that contest problems can be solved quickly using a "trick" (as well as via a longer approach), so knowing a range of shortcuts and being able to quickly identify the type of problem will improve speed. Look for shortcuts to problems that have an obvious but lengthy solution (e.g., writing out a sequence of numbers between 1 and 1000), but don't waste time looking for shortcuts when you can just quickly bang out the answer by "brute force." It is beyond the scope of this document to comprehensively describe common shortcuts, but a few examples are provided. Many of the shortcuts will arise when working through topic areas and practice tests, which is again reason for broad exposure to problems. One example of a shortcut would be divisibility rules. Rather than dividing a number repeatedly by different divisors to check for divisibility, one can apply quick divisibility rules. For instance, to check that the number 692 is divisible by 3, simply add the digits of the number (6 + 9 + 2 = 17) and check if that sum is divisible by 3. In this case 17 is not divisible by 3, so 692 is not divisible by 3 (further, when divided by 3, it has a remainder of 2: 17/3 = 5 R2). Another example would be the use of the prime factorizations of two numbers to determine the greatest common factor (GCF) or the least common multiple (LCM). The LCM is the union of the prime factorizations, while the GCF is the intersection of the prime factorizations.

A couple of additional tips will help with speed. Watch for clues in the problem statement as to the nature of the answer (e.g., if the problem says "state your answer as a reduced common fraction," that is a hint that the answer is likely not an integer). When working with an expression, be sure to "simplify before you multiply." There is no point in calculating the numerator and the denominator then dividing for an expression like 8! / 6! when it can be simplified to $8 \cdot 7$.

Basic Math

Basic math is the foundation for more complex concepts, so students should have a firm grasp in this area. Here, basic math refers to understanding our decimal number system, counting, types of numbers (natural, whole, integers, real, rational, irrational, etc.), addition/subtraction, multiplication/division, order of operations (PEMDAS), associative/distributive, proportions, fractions, etc. It is suggested that the Math Club coach cover definitions and vocabulary when addressing this topic. For example, explain what constitutes a natural number (1, 2, 3, … <but not 0>) or a common (improper) fraction (e.g., $^3/_2$). Vocabulary is important for translating word problems to numerical expressions. Describe what words translate to what operations ("total", "all together" = add; "difference", "exceed" = subtract; "of", "product" = multiplication; "split into", "share between" = divide, etc.). Discuss common qualifiers (positive, distinct, between, etc.) used in word problems.

Problem	Answer & Solution/Hints	Source
What is the quotient when 492 is divided by 3?	Simple division: $492 \div 3 =$ **164**	4 mic 2008, i-5
Evaluate: Two-thirds times two-thirds	Simple multiplication/comprehension of multiplying versus adding fractions $^2/_3 \times {}^2/_3 =$ **$^4/_9$**	4 mic 2008, cb-2.7
What is the product of 98 times 76?	Simple multiplication. Perhaps quicker than the standard algorithm would be: $100 \times 76 - 2 \times 76 = 7600 - 152 =$ **7448**	4 mic 2007, i-23
Put the following five values in order from smallest to largest. Your answer should consist of five letters in the correct order. $A = \frac{1}{4}$ $B = \frac{1}{2}$ $C = {}^1/_5$ $D = \frac{3}{4}$ $E = \frac{3}{8}$	Deals with nature of fractions (bigger denominator → smaller number). C & E may give the most uncertainty. But if one realizes that $3/8 > 1/4 = 2/8$, then it should all fall into place as: **CAEBD**	4 mic 2007, t-02
Find the difference between 100001 and 999	Simple subtraction, but can approximate & refine: $100000 - 1000 = 99000$ Then add 2: $99000 + 2 =$ **99002**	6 mic 2008, i-01
Cedric scores a goal 60% of the time. If he takes 20 shots, how many goals can he expect to make?	Translate to a multiplication problem: $0.6 \times 20 =$ **12**	6 mic 2008, mm-1.2
Express 11011_2 as a base 10 number.	$1 + 2 + 8 + 16 =$ **27** Remind students what place value means and how it can be used in such "base" problems.	5 mas 2004, i-31
Evaluate: $(3^{83} - 3^{80}) \div (9^{41} - 9^{39})$	$= (3^3 \cdot 3^{80} - 3^{80}) \div (9^2 \cdot 9^{39} - 9^{39})$ $= (3^3 - 1)(3^{80}) / (9^2 - 1)(9^{39})$ $= (26)(3^{80}) / (80)(3^{78})$ $= (13)(3^2) / 40$ $= (13 \cdot 9) / 40$ $=$ **$^{117}/_{40}$**	8 mas 2006, i-36

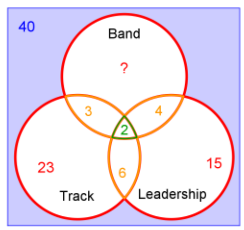
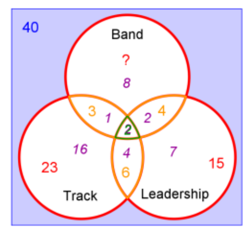
Number Theory

Number theory (perhaps referred to as "number sense") concepts deal with the relationships between numbers. Understanding how one number is "built" from other numbers can give one a better grasp of addition/subtraction and multiplication/division, particularly when done mentally. The relationships between numbers can also provide a basis for a quick approach to solving problems, in contrast with guess-and-check or brute force type approaches. Key aspects of number theory include prime (and composite) numbers and factors (with the associated concepts of multiples and divisibility). If one thinks in terms of factors (especially the prime factorization of a number), it is often straightforward to reach a solution. Prime factorization comes into play when determining the least common multiple (LCM), the greatest common factor (GCF), the number of factors a number has, the sum of a numbers factors, etc. Problems like the chickens & cows in a field or where two people go up a set of stairs but take a different number of steps at a time are solved with factors/multiples. Solving problems like "what is the units digit of 278^{278}?' involve patterns associated with multiples of a number. There is a wide field of interest/study with respect to prime numbers because of the role they play in mathematics and in specific applications, like encryption. The Prime Curios website is an interesting resource for information and trivia on prime numbers.

In addition to primes/factors, there are other aspects of number theory that students should know. The number system and bases are important (discussed in Basic Math above), because that is the basis for how numbers (a string of symbols) are put together (and can be taken apart to solve a problem). Students should understand what a sequence (list of numbers) is and the common arithmetic and geometric forms of sequences. On occasion, students may encounter a more complicated sequence (i.e., subtract 3 from every 2nd [even] number and add 2 to ever other [odd] number). At higher levels (8th grade), it may be possible that students would encounter a series (the sum of a sequence), but this is more likely a higher math type question as it fits well with limits and convergence topics. The On-Line Encyclopedia of Integer Sequences is a handy reference for identifying the nature of sequences, although it is not necessarily easy to determine the formula "rule" for a given sequence from the information provided. I will also group into the number theory category problems that deal with ratios (multiples) and set problems (Venn diagrams). While teaching formal set theory may be a more advanced concept, at least thinking in terms of sets can be useful for certain types of problems (including some combinatorics type problems).

Problem	Answer & Solution/Hints	Source
What is the sum of the two largest 2-digit distinct counting numbers that are not multiples of 2, 3, or 5?	Go backwards from 99 by twos (i.e., odd numbers), inspecting numbers for divisibility by 3, and 5. Stop when you find two numbers not divisible by 3 or 5. 97 + 91 = **188**	4 mic 2008, i-32
In the phrase "MATH IS COOL," each letter stands for a different digit and each word stands for a number. If the product of all the digits is 0, what is the largest possible sum of MATH + IS + COOL?	Think place value: We don't need to know actual numbers, but want to maximize place value. Zero must be included as one digit (since product of digits = 0). Note that there are two letter "O" characters, thus one digit is used twice. Lay out the words and start assigning digits, starting with 9 and working lower, to the high place values and moving (right) towards the units digit. Give the Os a higher digit since it is used twice. C 9000 O 700 O 70 L 3 M 8000 A 600 T 50 H 2 I 40 S 0 The sum is **18465**.	4 mic 2008, t-6
What is the eleventh term of the sequence whose first three numbers are 3, 6, 9?	Arithmetic sequence; constant difference of 3, starting at 3. Thus, 3×11 = **33**.	4 mic 2008, mm-4.2
What is the sum of all the prime numbers greater than 30 and less than 39?	Students need to know prime numbers < 100. Here, they are 31 & 37, so the answer is **68**.	4 mas 2001, cb-3.6
What is the least common multiple of 12 and 15?	It is important to understand LCM and GCF. Here, the student could probably get away with solving by inspection to come up with **60**. More rigorously, get the prime factorization of both numbers ($2^2 \cdot 3$ and $3 \cdot 5$) and take the union ($2^2 \cdot 3 \cdot 5$) to get the answer. GCF is the intersection of the prime factorizations (3).	6 mic 2008, i-11
The ratio of boys to girls on the math team is 5 to 3. If they can get three more girls to join, the ratio of boys to girls would be ten to seven. How many kids are currently on the math team?	Students must understand part to whole fraction versus part to part ratio. Here there are blocks of 8 students initially and blocks of 17 after 3 girls join. So you need a multiple of 8 plus 3 that is a multiple of 17. Look at 34 first (fails), then 51 (works, 51 – 3 = 48 = 8·6). Answering the question that was asked: **48**.	5 mic 2008, i-33

Problem	Answer & Solution/Hints	Source
In a field of daisies, there are daisies with 21 petals, 34 petals, and 55 petals. Thirteen daisies are picked with a total of 435 petals. There are only 4 daisies with 55 petals. How many daisies have 21 petals?	Subtract out the 55 petal daisies: $435 - 4 \cdot 55 = 215$. Now do like a cows & chickens problem…Suppose all daisies are 21-petal daisies…that gives $21 \cdot (13 - 4) = 189$ petals. The difference is $215 - 189 = 26$. For every 21-petal daisy replaced by a 34 petal daisy, you gain 13 petals. So there are 26/13 = 2 34-petal daisies, and thus 9 – 2 = **7** of the 21-petal daisies.	6 mic 2008, i-34
What is the units digit of 278^{278}?	Since we want the units digit, we only need to worry about the units digit of the base. Taking the first few powers of 8, we see that there is a repeating pattern in the units digit of the result: 8, 4, 2, 6, 8, 4, … Since there are 4 numbers in the pattern, 278/4 = 69 R2 = 2 (mod 4). So the answer is the 2^{nd} number in the pattern, **4**.	6 mic 2007, i-25
Frank has cows and chickens in a field. There are 27 heads and 78 feet. How many cows are in the field?	Variations on this theme are virtually guaranteed, and can show up even in mental math. What if they were all chickens? Take number of heads times 2 feet per animal (smaller number of feet/animal): $2 \times 27 = 54$. Subtract this from total number of feet: $78 - 54 = 24$. We have 24 feet too few. For every chicken (head) replaced with a cow, we get two more feet. So, divide 24 by difference in number of feet per animal: $24 / (4 - 2) = 12$, which is the number of cows needed to get the correct number of feet. Thus there are **12** cows and 27 – 12 = 15 chickens.	5 mas 2007, i-14
Alex the koala bear is climbing up a three-hundred foot cliff-face. Every day, he goes up six feet, and every night he slides down two feet. How many nights does he spend on the cliff-face?	Watch the bookkeeping with this sort of problem. The net change per 24 hours is 6 – 2 = 4. So 300/4 = 75. However, Alex actually makes it to the top on day 75 (+6) and never slides down during the night (-2), so he only spent **74** nights on the cliff face.	7 mic 2009, cb-1.1
What is the largest factor that the product of any set of three consecutive positive integers is certain to have?	Stop and think about this one in terms of (prime) factors. Within any group of 3 consecutive numbers, at least one number is even (i.e., has a factor of 2) and exactly one number will be a multiple of 3. If you start writing the prime factorization for the first several numbers, you'll see that the largest factor that any arbitrary group of 3 consecutive numbers will have is $2 \cdot 3 =$ **6**.	8 mic 2009, i-29
x, y, and z are prime numbers. When averaged, the result, q, is another prime number. What is the lowest possible product of x, y, z, and q?	Since we want the lowest product, start looking at combinations of small prime numbers. The sum of the 3 prime numbers must be a multiple of 3. 2+3+5 = 10 (fail). 2+3+7 = 12 (fails because average, 4, is not prime). 3+5+7 = 15 (works; average = 5, although question could be misread as "another" meaning "distinct"). So, $3 \cdot 5 \cdot 7 \cdot 5 = 21 \cdot 25 =$ **525**.	7 mas 1999, t-4

Problem	Answer & Solution/Hints	Source
Mathland Middle School's math team members all participate in at least one of three other school activities: band, leadership, and track. Of the 40 members of the team, 15 are in leadership, 23 are on the track team, 4 are in both leadership and band, 6 are in both leadership and track, 3 are in band and track, and 2 students are in all three activities. How many students are in the band, total?	It should be promptly clear that this is a Venn diagram problem, which applies algebra and bookkeeping skills. Solving this problem using a diagram is helpful, so start by making a diagram like that shown to the left. Keep track of the areas to which the numbers apply. The red numbers are for the entire red circle, orange numbers are for the entire intersection of two circles, green number is for the intersection of all three circles, and the blue number is the total population. Here, we can start putting purple numbers (right figure) into each area bounded by lines to show the number of people within that boundary only. Add up all the purple numbers except "band only," then subtract from the total number to get the band only number. Then add up the purple numbers within the Band circle: 1 + 2 + 2 + 8 = **<u>13</u>** students in Band.  	5 mas 2004, i-34

Algebra

Algebra, the manipulation of unknown variables and constants in equations, is a key component for addressing many math problems. If there is some fearful connotation implied by the word "algebra," consider that most elementary school students are doing a simple form of algebra routinely in problems like: 3 + <u> ? </u> = 8. In such problems, the words algebra and variable are not ever used, but in fact that is just what the student is applying: solving an equation for an unknown. While the field of algebra is broad and covers a number of aspects (gee–there are two whole years of "algebra" coursework in middle/high school), the types of math problems encountered generally apply basic algebra concepts. That is, one would only infrequently encounter a problem about factoring a cubic polynomial or finding roots of an equation. Some concepts, like the difference of two squares, $a^2 - b^2 = (a - b)(a + b)$, or the equation of a line, $y = m \cdot x + b$, *are* often useful. But there are two general aspects that students should be very comfortable with: translating from words to equations and solving (manipulating) equations.

One of the things that could make word problems challenging is the process of figuring out how the words translate into mathematical expressions, to which the student can apply the rules of math to arrive at a solution. One part of this process is understanding the vocabulary, as mentioned above in the Basic Math topic. It takes practice to identify the key words that translate to mathematical expressions, to identify (and ignore) extraneous information, and to figure out what it is that the problem is really saying/asking.

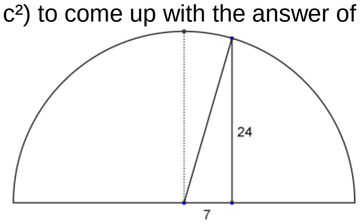
The mechanics of solving one or more equations for an unknown can be readily imparted to students as a set of standard rules and approaches. However, practice applying the process is important for solidifying understanding and increasing speed/efficiency. Equations are the basis for solving algebra problems, so describe how an equation is "balanced" and that what you do to one side, you must do to the other. Other aspects of manipulating equations are understanding what a "term" is and when you can add/subtract versus multiply/divide. The basic math principles of the distributive (and commutative/associative) property are important for knowing how one can combine like terms and rearrange terms in the equation. At least two approaches can be taught for solution of multiple equations, keeping in mind that one needs as many equations as there are unknowns. The substitution method is perhaps the most common approach (solve one equation for a particular variable, then substitute that expression in the other equation in place of that particular variable). In some cases, a quicker route to a solution is via the subtraction (or addition) of the equations, where one of the variables may "fall out" after subtraction or alternately the coefficients/factors may make a solution apparent.

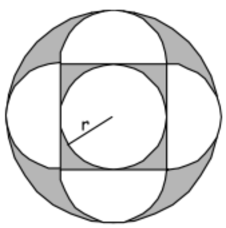
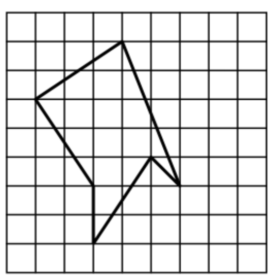
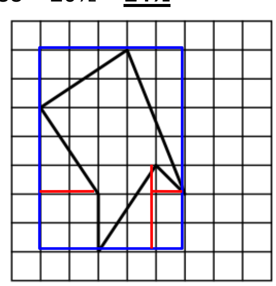
Problem	Answer & Solution/Hints	Source
If 20 – x = 15, then what is 3x?	Put the numbers all together and the variables all together. Add "x" to both sides and subtract 15 from both sides to get 5 = x. Then multiply, 3·x = 3·5 = **15**.	4 mic 2008, i-29
Aly is twice as old as her brother today. When she is 25 years old, her brother will be 20 years old. How many years old is Aly today?	Translate the words to equations: A = 2·B A' = 25 = 20 + 5 = B' + 5 Thus, A = B + 5 = 2·B and, 5 = 2B – B = B so, A = 2·5 = **10**	4 mic 2007, i-30
Grace swam h events at p swim meets in 2007 and swam p events at h swim meets in 2008. What is a simplified expression using h and p representing the average number of events Grace swam at a swim meet during 2007 and 2008?	Average is the total quantity divided by the total number: (h·p + p·h)/(p + h) = **(2ph)/(p + h)** This could be considered a bit vague; the problem should really say something like "…h events at each of her p swim meets in 2007…"	6 mic 2008, i-39

Geometry

Geometry is a broad field, but math contests don't typically stray into anything too wild. Indeed, even common aspects like constructions and proofs are uncommon (if not nonexistent). Most geometry pertains to standard shapes and the calculation of distance (length), area, and volume. However, geometry can be an ancillary component of other types of problems, such as a probability problem. Conversely, one may solve a geometry problem by applying other concepts (e.g., combinatorics). Encourage students to "see what isn't drawn" and be careful of appearances in figures (i.e., figures that are not-to-scale).

Students should be familiar with basic principles and common 2D and 3D shapes (and their associated formulas). Geometry basics include points, lines (straight, curved – ties in with graphing and algebra), line segments, angles (acute, obtuse, right, straight, complementary, supplementary, opposite), similarity (scaling), symmetry, concentricity, and inscribed/circumscribed. Circles (radius, diameter, chord, sector, arc, circumference, area) and triangles (acute, obtuse, isosceles, equilateral, right, base, altitude, area, vertex) are basic building-block shapes, or perhaps sub-components of larger shapes, and students should be comfortable with the associated terminology and formulas. Triangles are particularly useful and knowing the Pythagorean Theorem and special right triangles (45°, 30°-60°90°, 3-4-5, 5-12-13, etc.) can lead to speedy solution of a problem. Rectangles (including squares) would be the next most important shape to be familiar with. Other polygons (often regular) are routinely used in math problems, so students should be familiar with the polygon names and how to determine the interior angle of any polygon. Three dimensional shapes (sphere, cube, prisms, pyramids, cone, frustum of a cone, polyhedra) are also not uncommon objects of math problems, but moreso at higher grade levels. Through middle school level contests some lesser-taught material, like Heron's formula, is useful, but do not worry about seeing problems that involve things like trigonometric functions (sin, cos, tan).

Problem	Answer & Solution/Hints	Source
What is the area, in square feet, of a square with side length of 5 feet?	All sides are equal in a square. Area of a rectangle or square is the product of two adjacent sides. Thus, $5 \cdot 5 = 5^2 =$ **<u>25</u>**	4 mic 2008, i-3
If A is a square with a side length of 10 inches, and B is a circle with diameter 10 inches, which has the larger area, A or B? Answer "same" if the areas are equal.	Compare s^2 vs. $\pi \cdot r^2$ ➔ 100 vs. ~$3 \cdot 25 = 75$. Clearly the square (**<u>A</u>**) has the larger area.	4 mic 2007, i-11
What is the volume of a sphere with diameter of 2π, in un^3?	Volume of a sphere is $(4/3) \cdot \pi \cdot r^3$. $r = ½D$, so… $V = (4/3) \cdot \pi \cdot (\pi)^3 =$ **<u>$(4/3)\pi^4$</u>**	6 mic 2008, i-29
What is the supplement, in degrees, of a 42 degree angle?	Supplementary angles add up to 180° (complementary angles add up to 90°). So the supplement is 180 – 42 = **<u>138°</u>**	6 mic 2008, mm-4.4
As an ordered pair (x, y), name the point that is 4 units up and 2 units to the right of the point (6, 0) on a coordinate grid.	Add the "up" movement to the y coordinate and the "right" movement to the x coordinate to get (6+2, 0+4) ➔ **<u>(8, 4)</u>**	6 mic 2007, i-16
How many degrees does the hour hand of a clock travel from 8:00 AM to 10:30 PM the same day?	It is useful to know the rate of movement of the hands on a clock. The minute hand moves 6° in 1 minute. The hour hand moves 6° in 12 minutes. Here, it is useful to think a little broader. The hour hand has moved 2 hours worth (60° since each hour it moves 1/12th of 360°) plus half-an-hours worth (15°), for a total of **<u>75°</u>**.	6 mic 2007, i-23
The entrance to a railroad tunnel in the Cascade mountains is in the shape of a semi-circle. Seven feet from the center of the tunnel, the height of the entrance is 24 feet. How tall is the tunnel at its center, in feet?	Start by drawing a diagram to visualize the situation (see below). Given this diagram, several things should be apparent. The provided distances can be connected to form a right triangle. The hypotenuse of the right triangle is the radius of the semi-circle, which is the height of the tunnel at the center. The right triangle is a "special" triangle, namely a 7-24-25 triangle. Thus, one does not need to (but could) apply the Pythagorean theorem ($a^2 + b^2 = c^2$) to come up with the answer of **<u>25</u>**. 	5 mas 2004, i-35

Problem	Answer & Solution/Hints	Source
What are the coordinates, in (x, y) form, of the midpoint of the line segment with endpoints at (4, -5) and (-3, 9)?	The midpoint of a line segment is at (average of the x coordinates, average of the y coordinates) → **<u>(½, 2)</u>**. (Be careful about using decimal answers instead of fractions. In Math Is Cool, answers should all be in fraction form unless a decimal answer is asked for, although you might get a lenient scorer in particular cases).	7 mic 2005, i-26
What is the circumference in inches of a circle with an area of 64π square inches?	Circumference = $C = \pi \cdot D = \pi \cdot (2r)$ (The equation for circumference is derived from the definition of π as the ratio of the circumference to the diameter). Area = $\pi \cdot r^2$, so $r = \sqrt{64\pi/\pi} = 8$. Then C = **<u>16π</u>**	8 mas 2007, mm-3.3
In the figure, the semicircles are all congruent with radius r, and are tangent to adjacent semicircles, to the inscribed circle (which also has radius r), and to the circumscribed circle. Find the shaded area in square units if r = 3. 	The outer circle is radius 2r, thus has an area: $A_{outer} = \pi \cdot (2r)^2 = \pi \cdot (2 \cdot 3)^2 = 36\pi$ There are a total of 3 small circles within the outer circle (if you combine two semi-circles into one circle), which have a combined area of: $A_{unshaded} = 3 \cdot [\pi \cdot 3^2] = 27\pi$ Thus the shaded area is the difference: $36\pi - 27\pi =$ **<u>9π</u>**	7 mic 2006, pr-2
What is the number of square units in the area of the hexagon shown? Answer as a mixed number. 	Use the "draw what isn't there" approach to draw a box around the hexagon plus some additional lines to break the zone between the hexagon and the surrounding rectangle into right triangles and small rectangles. Then simply calculate the area of the outer rectangle and subtract the sum of the areas of the right triangles and small rectangles. $A_{hexagon} = 5 \cdot 7 - (½ \cdot 2 \cdot 3 + ½ \cdot 2 \cdot 5 + ½ \cdot 2 \cdot 3 + ½ \cdot 2 \cdot 3 + ½ \cdot 1 \cdot 1 + 2 \cdot 1 + 2 \cdot 2)$ $A_{hexagon} = 35 - (3 + 5 + 3 + 3 + ½ + 2 + 4)$ $A_{hexagon} = 35 - 20½ =$ **<u>14½</u>** 	8 mic 2008, pr-4

Statistics

In elementary/middle school math contests, the topic of statistics is primarily about the mean, median, and mode. Contests do not delve into other aspects such as standard deviation, error propagation, Gaussian distributions, etc. Students should be comfortable with the definitions of mean (average), median, and mode, because these concepts will arise explicitly or implicitly in problems. The presentation of data is another aspect that fits with this topic, whether it is via stem-and-leaf plot, bar charts, scatter graphs, or tables of data. Students must understand how to read charts and tables to extract the information of interest.

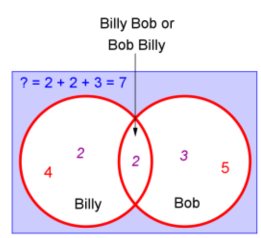
Problem	Answer & Solution/Hints	Source
The mode of a set of values is the value that occurs most often. Jackie kept a record of how many dog biscuits her dog Zero ate each day for a week. She wrote: Sunday, 4; Monday, 7; Tuesday, 3; and Wednesday, 5. Jackie lost her records for the rest of the week, but she remembers that the average was 4 biscuits per day and that Zero ate at least one dog biscuit each day. What was the mode of all seven of Jackie's recorded values for the week if there was only one mode? Give all possible answers.	Known numbers are: 3, 4, 5, and 7. Use the average to get the total number for the last 3 days: $4 \cdot 3 = 12$. Now, we need to partition 12 into 3 parts such that there is a unique mode. 1, 1, 10 1, 3, 8 1, 5, 6 2, 2, 8 2, 4, 6 2, 5, 5 3, 3, 6 4, 4, 4 So the possible modes are: **1, 2, 3, 4, and 5**. Looking at the key for this test, you'll note that the answer is given as "3, 4." As far as I can see, my answer meets all the criteria of the problem. So that must be an error in the answer key (but feel free to point out any flaw in my logic).	4 mic 2008, t-7
Sashawnda received an 87%, 93%, 94%, and an 80% on four tests. What is the minimum percentage Sashawnda must get on the next test to have an average of 90% over the five tests?	$(87 + 93 + 94 + 80 + x) / 5 = 90$ $354 + x = 90 \cdot 5 = 450$ $x = 450 - 354 =$ **96**	4 mic 2005, i-35
Find the mean, or average, of the following set of numbers: 1, 6, 8, 3, 0, 6, 4, 2, 7, 3	There are 10 numbers. Add the numbers and divide by 10. $(8 + 2) + (6 + 4) + (7 + 3) + (6 + 1 + 3) = 40$ $40/10 =$ **4**	4 mic 2006, i-19
What is the median of 150, 212, 72, 84, 515, 160?	The median is the number in the middle. For an even quantity of numbers, take the average of the two middle numbers. Here, the two middle numbers are 150 & 160, so the median is $(150 + 160) / 2 =$ **155**.	5 mic 2006, i-20
Find the median number in this stem-and-leaf plot. 0| 1 1 3 4 6 7 1| 1 3 6 8 9 2| 0 9 3| 2 3 3 3 4 6 7 4| 4 4 5 6 7	This is a straightforward problem, if you know how to read a stem-and-leaf plot. Here the tens digit is written in the leftmost column and a series of units digits are written out to the right. Thus, the middle row would represent the numbers 20 and 29. To find the median, cross off an equal quantity of numbers starting from the front (i.e., 1) and the back (i.e., 47), until you are left with one un-paired number of **29**, which is the median.	6 mic 2010, i-30
If 39 is the sum of three consecutive positive numbers, what is the smallest of the three numbers?	Treating the sum as the "total points" of an average will get you in the right ballpark after dividing by the number of numbers. Then you can adjust for the specific conditions (odd/even/sequential consecutive numbers). Here, divide 39 by 3 (because it was a sum of 3 numbers) to get 13, which is the middle number. The smallest number is thus **12** (largest is 14).	6 mic 1998, cb-2.7
The sum of three consecutive odd numbers is 111. If the median of the three is equal to x, what is 4x + 7?	Divide 111 by 3 to get 37, thus $x = 37$. $4x + 7 = 4 \cdot 37 + 7 = 111 + 37 + 7 =$ **155**	7 mas 2004, i-13

Combinatorics

The topic of combinatorics deals with counting and combinations/permutations, i.e., finding the number of configurations that satisfy a given set of rules (constraints). Combinatorics can be applied in many situations (not all of them obvious) and deals with arrangements or groupings (with or without replacement as well as symmetry of counting (as in Pascal's triangle and the binomial coefficient). Counting also includes the concept of bookkeeping, which implies keeping track such that items are not counted multiple times. For example, given the set of consecutive numbers from 10 to 70, the range is 60 (= 70 –10), but the number of numbers is 61 (= 70 – 9). Combinatorics is intertwined with probability, since probability is often based on counting "how many outcomes" versus how many total possible ways and probability also considers the permutations that may occur (e.g., in flipping a coin twice, I can flip a Head and a Tail or a Tail and a Head).

In addition to knowing the formulas for permutations and combinations (with and without replacement), there are several typical types of problems with which students should be familiar: number of ways to form a committee, number of ways to arrange items, number of possible license plate numbers, number of handshakes, and number of possible outfits. Test writers tend to try to find variations from these common problem types, particularly at the higher grade levels. For example, instead of how many ways can students stand in a line, a problem might ask 'how many ways can students sit around a circular table?'.

Problem	Answer & Solution/Hints	Source
In a soccer tournament with eight teams, how many games must be played if each team is to play every other team twice?	This is like a "handshake" problem. The first team plays 7 other teams, the second team plays 6 other teams (because we don't double count the game played against the first team), the third team plays 5 other teams, and so on. So, $7 + 6 + 5 + 4 + 3 + 2 + 1 = (8 \cdot 7) / 2 = 28$ However, each team played 2 games with each other team so the answer is $2 \cdot 28 =$ **56**. It wouldn't hurt students to know the first several triangular numbers (1, 3, 6, 10, …).	4 mic 2008, i39
How many ways can you arrange the letters in the word "PASSPORT"?	This is a permutation problem because order matters. Here, there are 8 letters, taken 8 at a time, with repeated "S" and "P" letters. The formula for permutations of n objects taken r at a time is $P(n, r) = {}_nP_r = n! / (n - r)!$ With repeated (identical) objects, you also divide by the number of repeats factorial. So here we have: $\frac{8!}{(8-8)! \cdot 2! \cdot 2!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 2} = 2 \cdot 5040 =$ **10080** Three things to note: (1) students should know factorials up to 7! at least, (2) simplify before you multiply, and (3) for completeness the formula for combinations is $C(n, r) = {}_nC_r = n! / [(n - r)! \cdot r!]$	6 mic 2008, cb1.6
Barbara has five blouses, four skirts, and ten pairs of shoes. How many different outfits can she make?	There are 5 choices for blouse, 4 choices for skirt, and 10 choices for shoes so the total number of possible outfits is $5 \cdot 4 \cdot 10 =$ **200**.	5 mic 2008, cb2.2
How many ways can three people be seated in 8 chairs?	Permutation problem. $P(8,3) = 8! / (8-3)! = 8! / 5! = 8 \cdot 7 \cdot 6 \cdot 5! / 5!$ $= 8 \cdot 7 \cdot 6 =$ **336**	6 mic 1998, cb-2.8

Problem	Answer & Solution/Hints	Source
How many possible 7-digit phone numbers are possible if the first three digits must be odd and the last digit must be prime?	Denote how many choices you have for each digit in the number. There are 5 odd digits and there are 4 prime digits. So, there are 5 ways to have an odd digit, 10 ways to have any digit, and 4 ways to have a prime digit. The number of possible phone numbers is the product of the number of ways to choose each digit: $5 \cdot 5 \cdot 5 \cdot 10 \cdot 10 \cdot 10 \cdot 4 = 125 \cdot 4 \cdot 1000 =$ **500000**.	4 mic 2008, i33
There are five light switches in a room, each of which can either be on or off. How many different ways can the switches be positioned?	There are 2 ways to choose the position of each switch, so the number of ways for positioning the five switches is: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 =$ **32**	6 mic 2006, cb-2.6
Janet goes to the Mini-Subway shop to buy a sandwich with one kind of bread, one kind of meat, and one vegetable. The only kinds of bread available are white and wheat, the only kinds of meat are turkey and ham, and the only vegetables are lettuce and tomatoes. How many different sandwiches can Janet buy?	There are 2 choices of bread, 2 choices of meat, and 2 choices of vegetable. The number of possible different sandwiches is $2 \cdot 2 \cdot 2 =$ **8**.	5 mic 2007, cb-4.2
Each of Billy Bob's sons has either "Billy" or "Bob" or both in his name. Four sons have "Billy" in their names and five sons have "Bob" in their names. Exactly one son is named Billy Bob and exactly one son is named Bob Billy. The other sons are named just "Billy" or just "Bob." In how many ways could Billy Bob have assigned names to his sons?	Start by figuring out how many sons Billy Bob has, which is a Venn diagram (set) problem. As shown in the figure below, we have groups of Billy, Bob, or both names. Working through the Venn diagram (purple numbers for zone within each boundary, red numbers for entire circle), we know there are two sons with both names and thus determine that there are 7 sons. We know one son is named Billy Bob, so we have 7 ways (i.e., 7 sons) to pick a son and name him Billy Bob. Once that is done, there are 6 ways (sons) to assign the name Bob Billy. Then there are 5-choose-3 ways to name 3 of the remaining 5 sons Bob → $C(5,3) = = 5! / [(5-3)! \cdot 3!] = 5 \cdot 4 / 2 = 10$ The remaining 2 sons can only be named Billy. Hence there are $7 \cdot 6 \cdot 10 =$ **420** ways to name the sons as described. 	7 mas 2007, t-8

Problem	Answer & Solution/Hints	Source
Bobby has twenty-one refrigerator magnets as shown, with a letter of the alphabet printed on each one. He wants to make the phrase RUBBER BABY BUGGY BUMPER. Bobby sets aside one R and three Bs to be the first letter of each word. He then randomly selects five letters. What is the probability that he selects the five letters needed to complete the first word of the phrase? A B B B B B B E E G G M P R R R U U U Y Y	Eliminating the 3 Bs and one R leaves 17 letters. We need letters "UBBER," which we could draw in any order. There are 5! / 2! = 60 ways to arrange those 5 letters (accounting for the repeated B). Calculate the probability of drawing letters in the order "UBBER": $^3/_{17} \cdot {}^3/_{16} \cdot {}^2/_{15} \cdot {}^2/_{14} \cdot {}^2/_{13}$ Each of the 60 arrangements will have the same individual probability, so add the probabilities (same as multiplying the single probability by 60). Simplifying before we multiply, we end up with: $^{(3 \cdot 3)}/_{(17 \cdot 7 \cdot 13)}$ = $\underline{\mathbf{^9/_{1547}}}$	7 mic 2008, i-38

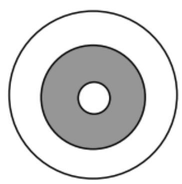
Probability

Probability pertains to the likelihood of the occurrence of an event, expressed as a fraction or decimal number ranging from 0.0 to 1.0 (or percentage ranging from 0% to 100%). Any number outside these bounds is an invalid probability. Probability can be determined in several ways. One method for ascertaining probability is from experimental data. One could, for example, perform an experiment with flipping coins, keeping track of the number of heads and tails. A probability that a coin will come up heads can then be determined from the experimental data as number of times heads came up divided by total number of flips. Experimental probability has some associated error stemming from the number of repetitions and/or measurement techniques, but should converge on the theoretical probability with enough repetitions. Experimental probability is less applicable to math contests, but may crop up in data-driven (table/chart) questions. Most commonly, probability will be calculated based on counting outcomes, as in number of favorable outcomes divided by total number of outcomes possible. For the coin flipping example, there is only one head on a coin (the "favorable" outcome), while there are two faces on the coin (number of total outcomes), so the probability of flipping a head is ½. As discussed in the Combinatorics section, such probability calculations rely on the ability to count the number of ways an outcome can occur, which may best be counted using a combinatorial technique. Another approach to calculating probability is based on a continuous property, such as area. For example the probability of a dart hitting a certain region of a dart board is proportional to the area of that region relative to the total dart board area.

Compound probability expresses the likelihood of multiple events occurring and can be proposed as having an "and" or an "or" relationship. The likelihood of multiple independent events occurring (i.e., event A and event B) is the product of their individual probabilities (i.e., the counting principle). The probability of event A <u>or</u> event B, where the events are mutually exclusive, is the sum of the probabilities for the individual events. Where events are not mutually exclusive, the overlap must not be double counted (i.e., subtract the probability of A and B).

Problem	Answer & Solution/Hints	Source
Katie has 3 red shirts, 5 blue shirts, and 3 orange shirts. If Katie picks one shirt at random, what is the probability that Katie picks an orange shirt?	There are 3 + 5 + 3 = 11 shirts total. The probability of picking an orange shirt is $\underline{\mathbf{^3/_{11}}}$ (3 ways of picking an orange shirt out of 11 ways of picking any shirt).	4 mic 2008, mm-2.2

<table>
<tr><th>Problem</th><th>Answer & Solution/Hints</th><th>Source</th></tr>
<tr><td>What is the probability of picking a red card from a standard deck of cards?</td><td>Students should be familiar with (fair) coins, (fair) dice and a standard deck of cards. A standard deck of cards has 52 cards in 4 suits (2 red, 2 black) with card values of Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King.
The probability of selecting a red card is the number of possible red cards divided by the total number of cards: 26 / 52 = <u>½</u></td><td>4 mic 2008, mm-1.3</td></tr>
<tr><td>If Hannah rolls a six-sided die and flips two coins, then how many possible ways exist of getting an even number on the die and 1 head and 1 tail from the coins?</td><td>This is looking for a combined probability of independent events, so we will multiply the individual event probabilities. There are 6 possible outcomes when rolling a single 6-sided die, 3 of which are even numbers. The probability of getting a head on a coin flip is the same as getting a tail, which is 1 out of 2. So the combined probability is: $^3/_6 \cdot ½ \cdot ½ =$ <u>⅛</u>.</td><td>4 mic 2008, i-16</td></tr>
<tr><td>Sixty-four unit cubes are formed into a cube, and then the outside is painted red. If one unit cube is chosen at random and is rolled, what is the probability that the top face is unpainted?</td><td>Visualize the cube, which is 4×4×4 (i.e., from the cube root of 64), and the categories of inside, outside face, outside corner, and outside edge. Get organized and list the numbers & probabilities:
<table>
<tr><th>Type of block</th><th># of blocks</th><th>P(pick this type</th><th># of red faces</th><th>P(rolling a red face)</th></tr>
<tr><td>inside</td><td>8</td><td>$^8/_{64}$</td><td>0</td><td>0</td></tr>
<tr><td>face</td><td>24</td><td>$^{24}/_{64}$</td><td>1</td><td>$^1/_6$</td></tr>
<tr><td>edge</td><td>24</td><td>$^{24}/_{64}$</td><td>2</td><td>⅓</td></tr>
<tr><td>corner</td><td>8</td><td>$^8/_{64}$</td><td>3</td><td>½</td></tr>
</table>
This is the combined probability of a set of exclusive sequential events. The probability of selecting a corner piece and rolling an unpainted face is ⅛ · ½ = $^1/_{16}$. Similarly we can calculate probabilities for edge (⅜ · ⅔ = ¼), face (⅜ · $^5/_6 = {}^5/_{16}$), and interior (⅛ · 1 = ⅛) blocks. The overall probability of picking a cube and rolling an unpainted face is the sum of these individual probabilities:
$^1/_{16} + {}^4/_{16} + {}^5/_{16} + {}^2/_{16} = {}^{12}/_{16} =$ <u>¾</u></td><td>6 mic 2008, t-3</td></tr>
<tr><td>The World Series of baseball, a famous sporting event, is played between two teams. As soon as either team wins 4 games, that team is declared World Champions (no game can end in a tie.) If a World Series is played between two teams of equal ability (so that each team's probability of winning any game is $\frac{1}{2}$), what is the probability that the World Champions are declared after only 4 games?</td><td>A team needs to win 4 games in a row, so we have the combined probability of:
½ · ½ · ½ · ½ = $^1/_{16}$
However, either team could win, so the answer is the sum of the probabilities for both cases:
$^1/_{16} + {}^1/_{16} =$ <u>⅛</u></td><td>6 mic 2003, i-39</td></tr>
<tr><td>What is the probability of drawing a spade or a four from a standard deck of 52 cards?</td><td>This problem demonstrates the need for bookkeeping because the events aren't mutually exclusive. You could draw a four of spades, which meets both conditions. The general formula for the probability of event A OR event B is:
P(A or B) = P(A) + P(B) – P(A and B)
Here, we have:
$^{13}/_{52} + {}^4/_{52} - {}^1/_{52} = {}^{16}/_{52} =$ <u>$^4/_{13}$</u></td><td>6 mic 2004, cb-4.8</td></tr>
</table>

<table>
<tr><th>Problem</th><th>Answer & Solution/Hints</th><th>Source</th></tr>
<tr><td>The probability that Helen will fall into the lake is one-fourth. The probability that Miya will fall into the lake is one-half. What is the probability that both Helen and Miya will fall into the lake, if these two events are independent?</td><td>The combined probability of independent events both occurring is the product of their individual probabilities.
P(A and B) = P(A) · P(B)
Here, we have:
$\frac{1}{4} \cdot \frac{1}{2} = \underline{\mathbf{\frac{1}{8}}}$</td><td>6 mic 2007, mm-3.2</td></tr>
<tr><td>What is the probability that if two dice are rolled, the sum of the top faces is five?</td><td>Make a table of sums, from which it is clear there are 4 ways to get a sum of 5 from two dice (accounting for all permutations). Thus, the probability is $^4/_{36}$ = $\underline{\mathbf{^1/_9}}$.
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr>
<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
</table></td><td>6 mic 2007, cb-2.7</td></tr>
<tr><td>Dart players are aiming at the bullseye as shown below. Given that all the dart players will hit the target with a dart, what is the probability their dart will hit the shaded area? The radius of the large circle is 5, the radius of the medium circle is 3, and the radius of the small circle is 1.
</td><td>The probability of hitting a given zone is proportional to the area of that zone. The areas are:
$A_{inner} = \pi \cdot 1^2 = \pi$
$A_{middle\_ring} = \pi \cdot 3^2 - A_{inner} = 9\pi - \pi = 8\pi$
$A_{outer\_ring} = \pi \cdot 5^2 - A_{middle} = 25\pi - 9\pi = 16\pi$
$A_{total} = \pi \cdot 5^2 = 25\pi$
The probability of hitting the shaded area is:
$^{8\pi}/_{25\pi}$ = $\underline{\mathbf{^8/_{25}}}$</td><td>5 mas 2004, i-38</td></tr>
<tr><td>There are humans and dogs on the lawn of a park. If you see 8 heads and 22 feet, what is the probability that two creatures selected at random will both be human?</td><td>First determine how many dogs & humans there are:
8·2 = 16
22 – 16 = 6
6 / (4 – 2) = 3 → 3 dogs → 5 humans
Now calculate the combined probability of two independent events (picking a human, picking a human). In the second event, the total number and the number of humans available are both decreased by 1 (since one was selected in the first event).
$^5/_8 \cdot {}^4/_7$ = $\underline{\mathbf{^5/_{14}}}$</td><td>8 mic 2007, cb-2.1</td></tr>
<tr><td>A standard die is rolled three times. What is the probability that a prime number is rolled on exactly one of the three rolls?</td><td>The probability that you roll a prime is ½ (primes are 2, 3, & 5) and the probability that you don't roll a prime is thus also ½. So the probability that you roll a prime, non-prime, and non-prime is the product of the three independent event probabilities:
$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$
However, there are 3 permutations that fit the criteria: P, N, N or N, P, N or N, N, P. So the total probability is the sum of the probabilities for each permutation:
$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \underline{\mathbf{\frac{3}{8}}}$</td><td>7 mic 2008, i-23</td></tr>
</table>

Rate Type Problems

Problems involving a rate (quantity per time) are actually a subset of algebra problems. However, they occur frequently enough to warrant a specific attention. Rate problems involve the equation rate = quantity/time → r = d/t, sometimes presented as d = r·t (hence the "drt" nomenclature). Rate problems are often associated with distance/speed, but really apply to any quantity per time (e.g., number of lawns mowed per hour). Solving a rate problem is often just a matter of applying the rate equation. Where two rates are involved (e.g., two cars driving at the different speeds for the same distance), it may be possible to eliminate one unknown that is common to both rates by solving for the variable, then setting the equations equal to each other (i.e., $r_1 \cdot t_1 = d = r_2 \cdot t_2$). Keeping track of units will help initially identify the useful form of the rate equation(s) and will keep students on track to solve the problem and answer the question that was asked.

Problem	Answer & Solution/Hints	Source
If the number of bacteria on a kitchen sink doubles every 5 minutes, then how many bacteria would there be in 15 minutes, given that there was 1 bacterium initially?	This is not strictly a "drt" problem, but it involves the rate of growth in terms of doubling time. Divide the total time by the doubling time: 15 / 5 = 3. Then double the initial quantity 3 times: $1 \cdot 2 \cdot 2 \cdot 2 = 1 \cdot 2^3 =$ **8**.	4 mic 2008, i-14
One fish can eat four bugs in one hour. At this rate, how many bugs can three fish eat in two hours?	Set the problem up to cancel units. The trickiest part may be recognizing how to represent the 4 bugs per fish per hour as a fraction. $\frac{4\text{ bugs}}{1\text{ fish} \cdot 1\text{ hour}} \times 3\text{ fish} \times 2\text{ hours} =$ **24** bugs	4 mic 2007, i-22
A train leaves Boston at 8am headed for Philadelphia at 80 mph. Half an hour later, a train leaves Philadelphia towards Boston on the same track at 70 mph. The track is 490 miles long. What time will the trains collide?	Train word problems—the stuff of Bart Simpson's (and others) nightmares. But really, one just needs to set up the problem with algebra/rate equations and solve it. Working in miles and hours: $r_1 = d_1 / t_1 \rightarrow 80 = d_1 / (t + 0.5)$ $r_2 = d_2 / t_2 \rightarrow 70 = d_2 / t$ $d_1 + d_2 = 490$ Rearrange the rate equations $80t + (80)(0.5) = 80t + 40 = d_1$ $70t = d_2$ Add the equations and substitute on the right hand side $150t + 40 = d_1 + d_2 = 490$ ‡ $150t = 450$ $t = 450 / 150 = 3$ hours (from 8:30 AM) So the trains will collide at **11:30 AM**. ‡A familiar student could jump to this equation by recognizing that the combined rate of movement is 70 +80 = 150 mph for some time t, the rate of movement is 80 mph for ½ hour, and the total distance traveled is 490. This equation shows the total distance traveled based on the rate and time is equal to the specified total distance.	6 mas 2006, cb-3.1

Problem	Answer & Solution/Hints	Source
Colin can mow a yard in one hour. Lee can mow the same yard in two hours. How long, in hours, will it take the two of them to mow the same yard working together?	Combine the efforts of the two boys. 1 yard/ 1 hour + 1 yard / 2 hours = $^2/_2 + ^1/_2 = ^3/_2$ yard/hour Move the number to the denominator (as the reciprocal) to get the time for one yard: 1 yard / (⅔ hour) So it takes the two boys **<u>⅔</u>** hours to mow one yard when working together.	6 mic 2003, mm-3.1
Spiderman, Garfield, and Shrek all enter a race as a relay team. Spiderman swings through the first mile in 3 minutes and 40 seconds. Garfield crawls the second mile in 116 minutes, and Shrek jumps the last mile in 20 seconds. What was their team's average speed in miles per hour? [Write answer as a decimal.]	The average rate of speed is the total distance divided by the total time. Total distance is 3 miles. Total time is 3⅔ + 116 + ⅓ = 120 min. 3 mile / 120 min. = $^3/_2$ mph = **<u>1.5</u>** mph	5 mas 2004, i-40
When empty and with the drain plugged, a certain pool takes 20 minutes to fill when only tap A is turned on and 12 minutes to fill when both tap A and tap B are turned on. When tap A is turned on but the drain is left unplugged, the same pool takes 36 minutes to fill. How many minutes would it take to fill the empty pool if only tap B is on but someone forgot to plug the drain?	This problem takes a little algebra. First, define some subscripts for Tap A (1), Tap B (2), and the drain plug (3). Then set up our rate equations (with the unknown volume,V, times, t_i, and rates, r_i). $t_1 = 20 = V / r_1$ $t_2 = 12 = V / (r_1 + r_2)$ $t_3 = 36 = V / (r_1 - r_3)$ $t_4 = x = V / (r_2 - r_3)$ Solve the first two equations for V and set them equal. $20r_1 = V = 12r_1 + 12r_2$ $8r_1 = 12r_2$ $r_2 = ⅔r_1$ Solve the first and third equations for V and set them equal. $20r_1 = V = 36r_1 - 36r_3$ $36r_3 = 16r_1$ $r_3 = ^4/_9\, r_1$ Substitute in for r_2 and r_3 into the 4^{th} equation and use the definition of the first equation. $x = V / (⅔r_1 - ^4/_9\, r_1) = V / (^6/_9 r_1 - ^4/_9\, r_1)$ $x = V / ^2/_9 r_1 = ^9/_2\, (V/r_1) = ^9/_2\, t_1 = (^9/_2)(20)$ x = **<u>90</u>** min.	8 mas 2006, i-37

Real Life and/or Physics-Based Problems

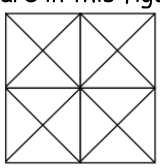
Depending on the contest (and the test writer), problems may be couched in the context of "real-life" situations or physics-based scenarios. Real-life situations are those that people would encounter in daily life as they shop, cook, do home repairs, drive, work, etc. People need to know how to make change, estimate the cost of discounted merchandise, perform unit conversions, calculate interest, and so on. In a related, but slightly different, category are problems that require some knowledge of how our world works. By this, I mean problems that deal with physics, chemistry, or biology. Students need to know about concepts like gravity, velocity, and simple machines. For example, a problem involving swimming upstream in a river implies that the river current is acting against the swimmer's propulsion, thus the net rate of movement is the difference between these velocities. A related example would be filling/draining a tank where the accumulation is the inflow minus the outflow. Another example is a lever (simple machine) in the form of a seesaw (teeter-totter), which entails a specific relationship between the length of the board on each side of the fulcrum to the force applied at the end of the board. Chemistry comes into play, for example, in terms of understanding concentration of a solute in a solvent (e.g., salt water). Biology could be discussed with respect to anatomy, how our bodies move, the nature of microbial growth, amongst many other aspects. The key point here is that students need to be aware of the world around them and have a basic understanding of science. Ultimately, as adults, the students will be applying math in their everyday lives and most likely in their careers, so it is important to connect the theory of math to the applications.

Problem	Answer & Solution/Hints	Source
If a wooden pencil costs 5 cents and a mechanical pencil costs 10 cents, then how many cents do 2 wooden pencils and 3 mechanical pencils cost?	Students must understand money and costs. 2 pencils × 5¢ + 3 pencils × 10¢ = **<u>40</u>**¢	4 mic 2008, i-7
How many total pints is 1 gallon, 3 quarts, and 1 pint?	Knowing your units is important. Here, we have: 8 + 6 + 1 = **<u>15</u>** pints.	4 mic 2008, mm-3.2
If it takes two and one-half cups of flour to make one batch of chocolate chip cookies, how many cups of flour will it take to make four batches of chocolate chip cookies?	This is a proportion problem in a cooking setting. 2 ½ c. per batch × 4 batches = **<u>10</u>** cups of flour	4 mic 2008, cb-1.2
An apartment currently rents for seven hundred fifty dollars a month. The monthly rent is expected to increase fifteen dollars every twelve months. What will the monthly rent be at the end of five years, in dollars?	Understanding the nature of money, payments, interest, etc. is important. This problem is slightly tricky in that the intent (not stated explicitly) is that the rent increases at the first of the year. So the first year is \$750/mo., second year is \$765/mo., etc. Thus, the answer is found as: 750 \$/month + \$15×4 = **<u>810</u>** \$/month	4 mic 2008, cb-1.7
When will it be 3 hours 46 minutes past 5:45 AM?	Students need to know how to work with time and to pay attention to AM/PM. Here, add 5 + 3 = 8 and 45 + 46 = 91 min. = 1 hr 31 min. So the final time is **<u>9:31 AM</u>**.	4 mic 2008, cb-2.8
[In this problem, use the facts that 1 ton = 2000 lbs and 1 mile = 5280 feet.] An elephant weighing 2.64 tons and a rabbit weighing 1 lb. are balanced on a very long, perfectly rigid teeterboard (seesaw). If the elephant starts sliding toward the fulcrum at the uniform rate of 1 foot per minute, how many miles per hour must the rabbit run in order to maintain balance?	This problem deals with a seesaw. Students should understand that the force required to lift a given load is inversely proportional to the distance from the fulcrum. That is, a small weight far from the fulcrum can balance a heavy weight near the fulcrum. Also note that weight is a force (in contrast with mass, which is a quantity of material). $f_1 \cdot d_1 = f_2 \cdot d_2 \rightarrow (2.64 \cdot 2000)d_1 = 1 \cdot d_2$ $5280d_1 = d_2$ Thus, for every foot decrease in d_1, the distance d_2 must decrease by 5280 ft (think factors/product). So the rabbit must run at 5280 ft/min to offset the 1 ft/min movement of the elephant. Now, do the unit conversions to answer the question that was asked: 5280 ft/min = 1 mile/min = **<u>60</u>** mph	4 mas 2003, i-37

Problem	Answer & Solution/Hints	Source
Biff and Eho are buying ice cream at Dairy Queen. Biff buys a Blizzard for $2.73. Eho buys a Brownie Earthquake for $3.19. Before they paid for the ice cream, they had three times as much money as they had left after they paid for the ice cream. How much money, in dollars, did they have before buying ice cream?	Another money problem. Students should be comfortable with "odd" names. Names are drawn from different ethnic backgrounds or may just be made up. Focus on the important parts of the problem statement to answer the question that is asked. $x = 3\cdot(x - 2.73 - 3.19) = 3\cdot(x - 5.92)$ $3\cdot 5.92 = 17.76 = 3x - x = 2x$ $x = 17.76 / 2 =$ $ **<u>8.88</u>**	4 mic 2007, i-35
In a river with a steady current, it takes a frog 20 minutes to swim a certain distance upstream, but it takes her only 10 minutes to swim back. How many minutes would it take a stick to float this same distance downstream?	This is a rate type problem that illustrates the combined velocity (added when stream & frog are going the same direction, subtracted when going in opposite directions). Rates of speed can similarly be combined in train problems to give the overall/net rate. $r_1 = d / 20 = r_{frog} - r_{river} \rightarrow d = 20r_{frog} - 20r_{river}$ $r_2 = d / 10 = r_{frog} + r_{river} \rightarrow d = 10r_{frog} + 10r_{river}$ Subtracting the second equation from the first, $0 = 10r_{frog} - 30r_{river}$ $30r_{river} = 10r_{frog}$ $r_{river} = \frac{1}{3}r_{frog} \rightarrow r_{frog} = 3r_{river}$ Plugging back into the second equation $d/(r_{frog} + r_{river}) = d/(3r_{river} + r_{river}) = d / 4r_{river} = 10$ $d / r_{river} = t_{river} = 4\cdot 10 = 40$ The time for a stick to float downstream, t_{river}, is just the distance divided by the rate of the river, so the answer is **<u>40</u>** min. Note that if you plug back into the first equation you get a negative number, but the sign just indicates the direction of movement.	4 mas 2005, i-39
Solution A is a solution of water and hydrochloric acid and is 70% water. How many liters of water should be added to 30 liters of solution A to create a solution with 20% hydrochloric acid?	This problem (indirectly) deals with concentration (quantity per volume) and mixing/dilution. Solution A has 70% water (thus 30%) acid, so out of 30 L of Solution A, 9 L are acid. In the new solution it is 20% acid so 9 L = $0.2 \times V_{new} \rightarrow V_{new} = 45$. The difference in volume is 45 – 30 = **<u>15</u>** L of water added.	7 mas 2000, mc-8
A 14-ounce mixture of sugar and salt is 10% sugar. How many ounces of sugar must be added to produce a mixture that is 60% sugar? Give your answer as a decimal.	Another concentration/mixture type problem. Currently there is 0.1×14 = 1.4 oz. of sugar (and thus 12.6 oz salt). $1.4 + x = 0.6\,(14 + x)$ $1.4 + x = 8.4 + 0.6x$ $x - 0.6x = 0.4x = 8.4 - 1.4 = 7$ $x = 7 / 0.4 =$ **<u>17.5</u>** oz.	7 mas 2006, pr-4

Miscellaneous Problem Types

Those who write tests are often looking for something novel to give a problem a twist or unexpected setting. Another approach is to introduce problems outside or at the fringes of "typical" topics (algebra, geometry, probability, etc.). The example problems in this section are a smattering of such atypical problems. Atypical/fringe topic problems will likely comprise a small percentage of a set of contest problems, but you can expect to see a few on every test. As long as a student isn't confused by the way the problem is presented, many of these can be solved using knowledge of the standard topic areas. Solution of some of these atypical problems requires specific knowledge (e.g., 'write this number in scientific notation'). An approach to address atypical problems is through student exposure to (practice with) many types of problems—the more they have been exposed to, the better prepared they are to solve similar problems or to apply their knowledge to new situations.

Problem	Answer & Solution/Hints	Source
If (a*b) = 2a + 2b, then what is ((1*2)*3)?	This is an example of defining an operator (here the "*", but other symbols can be used) that represents some functional relationship. This is solved by simply plugging the numbers into the equivalent mathematical formula. $(1*2) = (2\cdot1 + 2\cdot2) = 6$, then $(6*3) = (2\cdot6 + 2\cdot3) =$ **<u>18</u>**	4 mic 2008, i-23
If today is Sunday, then what day is it 69 days after tomorrow?	This can be solved using modular arithmetic (or by just dividing and finding the remainder). 69 = 6 (mod 7) So, count 6 days from tomorrow (Monday) to get the answer of **<u>Sunday</u>**.	4 mic 2008, i-27
How many triangles are in this figure? 	Get organized! Start with the smallest triangle and work your way up to bigger triangles. Use symmetry to speed counting. Look at the lines from all angles. Triangle size # 1 16 2 16 4 8 8 4 Total **<u>44</u>**	4 mic 2008, i-38
There are 2 bugs in a rug and 3 rugs in a jug. How many bugs are in 4 jugs?	This is like a unit conversion problem and it has alliteration that is distracting (particularly as a verbal question). There are 6 bugs in a jug, so 4 jugs has **<u>24</u>** bugs.	4 mic 2007, cb-4.3
Write 0.000072900 in scientific notation	Students should be familiar with scientific notation. **<u>7.29×10^{-5}</u>**	6 mic 2008, i-18
Madam Hippo has her own currency consisting of Hips, Haps, Hups, and Hops. 28 Hips is the same as 16 Haps. 21 Haps is the same as 7 Hups. 14 Hups is the same as 4 Hops. If one United States dollar is equal to 7/8 of one Hip, how much, in US dollars, is one Hop worth?	This is a unit conversion problem. Students need to understand how to arrange values (either multiplying [numerator] or dividing [denominator]) to convert from one unit to another. $\frac{1\ \$}{\frac{7}{8}\ \text{Hip}} \cdot \frac{28\ \text{Hip}}{16\ \text{Hap}} \cdot \frac{21\ \text{Hap}}{7\ \text{Hup}} \cdot \frac{14\ \text{Hup}}{4\ \text{Hop}} \cdot 1\ \text{Hop}$ = $ **<u>21.00</u>**	6 mic 2008, i-32
Express as a reduced common fraction: $0.\overline{27}$	Repeating decimals can be converted to a fraction by algebraic manipulation. let $x = 0.\overline{27}$, then $100x = 27.\overline{27}$ $100x - x = 99x = 27.\overline{27} - \overline{27} = 27$ $x = {}^{27}/_{99} =$ **<u>$^{3}/_{11}$</u>**	7 mas 2004, i-10
My rhino can do the rumba in the rain while drawing rhombuses. You try and impress me with your squirrel who can salsa in the sunshine while drawing squares. However, my question to you is does your squirrel draw rhombuses while doing the salsa in the sunshine?	Don't get distracted by the alliteration and extraneous information. Sort out the important information and solve the problem. The question here boils down to "Is a square a rhombus?" The answer is **<u>Yes</u>**.	6 mic 2008, i-7

Problem	Answer & Solution/Hints	Source
Two dozen widgets have a total cost of \$a.9b, where a and b stand for digits, not necessarily different. Each widget costs the same whole number of cents. What is the greatest possible cost, in cents, of each widget?	Note: widget = doodad = thingamajig Don't get confused by the terminology. We want the largest three digit number (cents) that is a multiple of 24 (two dozen) and has a 9 as a tens digit. We can quickly see that $40 \cdot 24 = 960$ and adding another 24 gives 984. You could count backwards by 24 until you find a number with a 9 in the 10s digit. Alternately, you could set up the problem in the standard algorithm form and use logic/basic math knowledge. $\begin{array}{r} 24 \\ \times\ yz \\ \hline htu \\ +jk \\ \hline a9b \end{array} \rightarrow \begin{array}{r} 24 \\ \times\ 3z \\ \hline htu \\ +72 \\ \hline a9b \end{array}$ Clearly, y must be less than 4, because $5 \cdot 2$ gives a 2-digit value (which would result in a 4-digit answer). And $y \neq 4$ (from our earlier multiplication, so let's assume y = 3. Then z has to be something that gives a tens digit (t) of 7 when multiplied by 24. You can make a table of values that represent the possibilities when multiplying by different z digits (accounting for the carry): $\begin{array}{rrrrrrrrrr} z & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 \cdot z & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 \\ 4 \cdot z & 4 & 8 & 12 & 16 & 20 & 24 & 28 & 32 & 36 \\ t & 2 & 4 & 7 & 9 & 2 & 4 & 6 & 9 & 11 \end{array}$ The only value of z that gives a 7 in the 10s digit is 3. So the answer is <u>**33**</u>¢.	8 mic 2007, i-21
I left my calculator out in the rain, and now it gives me weird answers to division problems. When I enter "175 ÷ 4", it shows the answer 433. When I enter "88 ÷ 3", it shows the answer 291. When I enter "315 ÷ 2", it shows the answer 1571. When I try to divide a certain number, n, by 5, it shows the answer 933. What is n?	This type of problem is not uncommon, where the student must figure out what the calculator is doing. It's sort of a combination logic/ number theory type problem. Here, one must determine the pattern of behavior to come up with the unknown number. Doing the three divisions given as examples to see what the quotient should be, it would hopefully become apparent that the calculator shows the quotient with the remainder tacked on as the last digit. Thus the answer would be $5 \cdot 93 + 3$ = <u>**468**</u>. If you don't see the readily see the pattern, this would probably be a problem to skip and come back to it later if you have time.	5 mas 2007, i-31
Three people (A, B, and C) are suspects in an investigation and made the following statements. If exactly one statement is true, who committed the crime, A, B, or C? A: B is innocent. B: I'm guilty. C: A is innocent.	It can be easy to get turned around or lost when thinking about logic problems. Here, being organized can help. Start by supposing that statement A is true. Then, statements B and C must be false (per the 'only one is true' criterion), which leads to A being guilty. If B is true, then A and C are false, which leads to a contradiction between statement B and the opposite (negation) of statement C. If C is true, then A and B are false, which leads to a contradiction between the opposite of A and the opposite of B. Therefore, A must be the true statement and person <u>**A**</u> is guilty.	8 mic 2005, i-27

Problem	Answer & Solution/Hints	Source
Give the letters of all of the following statements that are true. If no statement is true, answer "none." A) It is impossible to subtract a larger number from a smaller number. B) The diagonal of a square divides the square into two congruent equilateral triangles. C) When you divide one positive number (the dividend) by another positive number (the divisor), the answer (the quotient) is always less than the dividend. D) The counting numbers (= natural numbers = positive integers) do not include 0. E) When you double the radius of a circle, you double its area.	This is essentially a multi-part question presented in a manner where there is one overall answer. The questions, covering several topic areas, just need to be individually assessed. The only statement that is true is **D**.	5 mic 2008, t-3
If all piano players are poor, and Biff is poor, does it follow logically (necessarily) that Biff is a piano player? Answer "yes" or "no."	Students may not recognize logic as an aspect of mathematics (i.e., important in reasoning and proofs), so could be unprepared for such questions. This is fairly straightforward question. **No**, Biff is not necessarily a piano player, even though he is poor and it is asserted that all piano players are poor.	5 mas 2007, i-8

SELECTED RESOURCES

List of Math Competitions

The table below lists information about selected math competitions. Some competitions are in-person gatherings of students from multiple schools, while others are tests taken at your school and the results send in online (or answer sheets sent via mail). In addition to the contests listed, the Carmichael M.S. Math Team hosts more informal "Warm-Up" contests for middle school and elementary level students (in October and February, respectively). Also, the Columbia Basin College has administered a CBC Prize Exam for middle school students with some nice (tangible) incentives.

Selected Competition Opportunities [1]

Competition	Max. # Teams (# Students per Team)	Eligibility	# Meets / Schedule	# Events, Problems, & Duration (per Meet)	In Person ?	Cost [1]	More Information
Math Is Cool (MIC)	No Limit [2] (4), plus 2 alternates	4th - 12th grades, by grade	2 (R, S) 6: Feb. + May 7/8: Nov. + Dec.	6 events, 4-40 problems/event, 5-30 min./event	Yes	$10/grade + $40/team + $10 for PO	www.academicsarecool.com
MATHCOUNTS (MC)	1 (4) + 4 individuals	6-8th grades, combined	3 (R, S, N) Feb., Mar., May	3-5 events, 8-30 problems/event, ~3 hrs total	Yes	$90/team + $25/individ.	www.mathcounts.org
Washington State Math Olympiad (WAMO)	No Limit (3-4)	5th-8th grades, by grade	1 (S) May	6 events, 1-5 problems/event, 20-60 min./event	Yes	$40/team (no PO)	www.wsmc.net/olympiad
Washington State Math Championship (WAMC)	No Limit (4)	5th - 12th grades, by grade	1 (S) Apr.	6 events, 5-30 problems/event, 15-30 min./event	Yes	$45/team	www.blainesd.org/mathchamps /middle_school/Home.html
Mathematical Olympiads for Elementary & Middle Schools (MOEMS)	No Limit (35 max.) Team: top 10 scores	4-6th grades (Div. E) & 7-8th grades (Div. M)	5 monthly, Nov. - Mar.	1 event, 5 problems, 30 min.	No	$99/team (online data entry)	www.moems.org
Online Math League (OML)	1 per grade (4+) Team: top 4 scores	2nd - 8th grades ,by grade	3 Nov., Jan., Mar.	1 event, 15 problems, 30 min.	No	$99/grade level	www.onlinemathleague.com
Math League (ML)	1 per grade (5+) Team: top 5 scores	4th - 8th grades, by grade	1 Feb.	1 event, 40 problems, 30 min.	No	$40/grade level	www.themathleague.com
Continental Math League (CML)	1 or 2 per grade (6+) Team: top 6 scores	2nd - 9th grades Eucl. / Pyth. Divs.	5 monthly, Nov. - Mar.	1 event, 6 problems, 30 min.	No	$85/team (1st) $70/team (addtl.)	www.continentalmathematics league.com
AMC-8	1 / school (No Limit) Team: top 3 scores	6th - 8th grades, combined	1 Nov.	1 event, 25 problems, 40 min.	No	$35 + $12 per 10 individuals	amc.maa.org/e-exams/e4-amc08/amc8.shtml
Gauss	N/A (No Limit)	7th & 8th grades	1 May	1 event, 25 problems, 60 min.	No	$3/individ. + 15% S&H	cemc.math.uwaterloo.ca/ contests/gauss.html
Purple Comet (PC)	No Limit (1-6)	6th - 8th grades, combined	1 Apr.	1 event, 15 problems, 60 min.	No	FREE	purplecomet.org
Noetic Learning Math Contest	1 per grade (No Limit)	2nd – 5th grades	2 Oct., Apr.	1 event, 20 problems, 45 min.	No	$29/team	www.noetic-learning.com/mathcontest
Educontest / Mathfax (EDU)	1 per grade (3+) Team: top 3 scores	3rd - ~10th grades	4 Nov., Dec., Feb., Mar.	1 event, 25 problems, 30 min.	No	$80/grade level + $6 S&H	www.educontest.com
ABACUS International Math Challenge	N/A (No Limit)	3rd/4th (A), 5th/6th (B), & 7th/8th (C)	8 monthly Sep. - Apr.	1 event, 8 problems, No time limit	No	FREE	www.gcschool.org/program/ abacus/index.aspx
R = Regional, S = State, N = National							

[1] This information is current for the 2011-12 school year, although details may be approximate (if organizations have implemented changes from past years). Always check with the sponsoring organization for current details on competition dates, format, and costs.

[2] As of the 2011-12 school year, there are no longer limits on the number of teams for the MIC regional (Championships) events. Schools are still limited to one team at the state (Masters) level event; schools that qualify for MIC Masters are notified and are invited to register at/after the regional event.

Nominal Timeline for Registration & Contests

Register	MC (club) MOEMS Noetic ABACUS	MIC (R,7/8) CML OML AMC-8 EDU	MIC (S,7/8)	MC (R)	MIC (R,6) WAMC ML	WAMO MIC (R,5) MIC (S,6)	MIC (R,4) MIC (S,5) Gauss PC	MIC (S,4)	
Month	**Sep.**	**Oct.**	**Nov.**	**Dec.**	**Jan.**	**Feb.**	**Mar.**	**Apr.**	**May**
Compete	ABACUS	Noetic ABACUS	MIC (R,7/8) MOEMS CML OML AMC-8 EDU ABACUS	MIC (S,7/8) MOEMS CML EDU ABACUS	MOEMS CML OML ABACUS	MC (R) MIC (R,6) MOEMS CML ML EDU ABACUS	MC (S) MIC (R,5) MOEMS CML OML EDU ABACUS	MIC (R,4) WAMC PC Noetic ABACUS	MIC (S,4-6) WAMO Gauss MC (N)

Websites

There is a huge amount of material pertaining to mathematics available online. Material ranges in level from early learning to university and spans topics from humor to reference works. Included below are selected website resources that may potentially be useful. Note that you should evaluate the suitability of these websites for your purposes in terms of content level and any bias (i.e., in blogs) or commercial nature (i.e. in "instructional" websites). These links are provided as a starting point for exploration; a specific Internet search may yield more specific results appropriate for your needs.

Website	URL
Portals	
Portal:Mathematics (Wikipedia)	en.wikipedia.org/wiki/Portal:Mathematics
Mathematics -- Wikipedia	en.wikipedia.org/wiki/Mathematics
PlanetMath	planetmath.org
Wolfram MathWorld	mathworld.wolfram.com
Connexions	cnx.org/content/browse_content/subject/Mathematics%20and%20Statistics
The Mathematical Atlas	www.math-atlas.org
The Art of Problem Solving	www.artofproblemsolving.com
Math Open Reference	www.mathopenref.com
Books	
Free Mathematics eBooks	freebookcentre.net/SpecialCat/Free-Mathematics-Books-Download.html
Useful e-books	mate.cucei.udg.mx/ebooks/index_util.html
NCERT Text Books	ncertbooks.prashanthellina.com
Online Math Text books	people.math.gatech.edu/~cain/textbooks/onlinebooks.html
FreeScience Math books	www.freescience.info/mathematics.php
Reddit Math books	www.reddit.com/r/mathbooks
Mathematics – Free E-Books	www.e-booksdirectory.com/listing.php?category=3
Reference	
Encyclopaedia of Mathematics	eom.springer.de
Engineering Mathematics Handbook	www.maths.abdn.ac.uk/~igc/tch/engbook/engbook.html
Hdbk Math. Funct. (Abramowitz & Stegun)	www.math.ucla.edu/~cbm/aands
Bridge to Algebra chapters	wikitamath.wiki.nthurston.k12.wa.us/3rd+Period, wikitamath.wiki.nthurston.k12.wa.us/1st+Period
Inner Algebra	boostmath.com/mental-algebra
A Review of Basic Geometry	www.andrews.edu/~calkins/math/webtexts/geomtoc.htm
Trigonometric Delights	press.princeton.edu/books/maor
Dave's Short Course in Trigonometry	www.clarku.edu/~djoyce/trig
Forums	
Ask Dr. Math	mathforum.org/dr.math
MathOverflow	mathoverflow.net

Website	URL
Problems / Practice / Instruction	
Preparation Drills for MATHCOUNTS	mathcounts.saab.org/mc.cgi
AGMath.com	www.agmath.com
MasterMathMentor.com	www.mastermathmentor.com
Mathwords	www.mathwords.com
Math Resources List	www.khake.com/page47.html
Blogs	
Let's Play Math! (blog)	letsplaymath.net/2008/09/05/math-club-plans
Neatorama	www.neatorama.com/tag/math
MATHCOUNTS Notes	mathcountsnotes.blogspot.com
Math With Dad	mathwithdad.blogspot.com
Math Blog	www.mathblog.dk
MAA NumberADay	maanumberaday.blogspot.com
MAA MinuteMath	maaminutemath.blogspot.com
Tanya Khovanova's Math Blog	blog.tanyakhovanova.com
Ramblings of a Math Mom	mathmomblog.wordpress.com
Wild About Math!	wildaboutmath.com
Starting a Math Club	
AMC Math Club	amc.maa.org/mathclub/index.shtml
MATHCOUNTS Club Program	mathcounts.org/page.aspx?pid=221
Texas State University, Suggestions for Starting a Math Club	www.txstate.edu/mathworks/Additional-Resources/kidsmath/clubsuggestions.html
Washington Student Math Association, Math Club Starter Package	www.wastudentmath.org/content/clubs/plan/StarterPackComprehensive.pdf
Ann Perry, “Math Club Starting in Kindergarten”	www.nctm.org/publications/article.aspx?id=29197
Building a Successful MATHCOUNTS Program	www.artofproblemsolving.com/Resources/articles.php?page=mathcounts
Careers	
Math Matters, Apply It! (SIAM)	www.siam.org/careers/matters.php
Engineering, Science and Math Careers	www.khake.com/page53.html
WeUseMath	weusemath.org/?q=careers
History of Mathematics	
Math Quotes	www.onlinemathlearning.com/math-quotes.html
The Story of Mathematics	www.storyofmathematics.com
MacTutor History of Mathematics	www.gap-system.org/~history
The Saga of Mathematics	math.widulski.net/slides/slides.html
Famous Mathematicians	fabpedigree.com/james/mathmen.htm
Biographies of Women Mathematicians	www.agnesscott.edu/lriddle/women/women.htm

Books

There are a wide variety of books on math, from textbooks to contest preparation books. You are encouraged to visit your local bookstore or an online bookstore to peruse options that may be useful. The online retailers tend to list other related or suggested books based on your searches, so you will find plenty of material to explore. The book *Competition Math for Middle School* by Jason Batterson would be a good place to start an online search (it is also offered at the AoPS website discussed below). In addition to general searching online and in bookstores, we'll point to two specific types of resources. The first would be materials available from the organizations sponsoring math contests. Typically, such material will not be instruction on how to solve problems, but will include solutions for solving each problem. The second resource of interest is the Art of Problem Solving company (http://www.artofproblemsolving.com/Store/index.php), started by Richard Rusczyk a former participant in the national MATHCOUNTS competition. Among the variety of things they do, the AoPS company has developed a curriculum that is presented in a series of books that provide instruction and practice problems.

Problem Solving

A selection of books discussing problem solving from a multiple perspectives is listed below, in perceived order of usefulness. While a subset of these books have problems suitable for middle school or elementary level students, any one of them would be worthwhile reading material for coaches or teachers. Problem solving strategies are also discussed in an excerpt from a prior-year (but not recent) MATHCOUNTS Handbook, which is available at sites.google.com/a/g-cacegypt.org/cac-grade-7-math/problem-solving/MathCountsProblemSolvingStrategies.pdf.

Book	Comments
Immergut, B. 2003. *Master Math: Solving Word Problems*. Career Press, Franklin Lakes, New Jersey.	Not specifically about problem solving strategies, but is targeted at solving word problems. Covers equations, percents, age problems, mixing problems, measurement problems, rate problems, statistics/probability, and geometry. Good for elementary and middle school level students.
Posamentier, A.S., and S. Krulik. 2009. *Problem Solving in Mathematics: Grades 3-6*. Corwin Press, Inc., Thousand Oaks, California.	Discusses problem solving techniques (find a pattern, make a diagram, etc.) and provides useful discussion and example problems.
Wingard-Nelson, R. 2005. *Word Problems Made Easy*. Enslow Elementary, Berkeley Heights, New Jersey.	While this is ostensibly about word problems, it covers the typical problem solving techniques (draw a picture, make a list, work backwards, etc.). Suitable for lower grades (2^{nd} to perhaps 4^{th}).
Polya, G. 1973. *How to Solve It: A New Aspect of Mathematical Method*. Princeton University Press, Princeton, New Jersey.	This is a classic work (originally published in 1945) with a wealth of information and tactics. This book has influenced many subsequent works that are based on Polya's four-step process of understanding the problem, devising a plan, carrying out the plan, and looking back.
Krantz, S.G. 1996. *Techniques of Problem Solving*. American Mathematical Society, Providence, Rhode Island.	Takes a "learn by doing approach." Problems are approachable for HS or advanced M.S. students. Covers basics of problem solving, geometry, counting, probability, logic, algebra, and other material.
Posamentier, A.S. 1996. *The Art of Problem Solving: A Resource for the Mathematics Teacher*. Corwin Press, Inc., Thousand Oaks, California.	This is a collection of articles on problem solving that are intended to convey a "host of interesting and useful ideas" to mathematics teachers. Covers a variety of problem-solving techniques and topic areas.

Book	Comments
Zeitz, P. 2007. *The Art and Craft of Problem Solving*. John Wiley & Sons, Inc., Hoboken, New Jersey.	The purpose of this book is "mathematical problem solving for college-level novices," so it is a bit more advanced. Covers strategies and tactics for investigating/solving problems, algebra, combinatorics, number theory, geometry, and calculus.
Tao, T. 2006. *Solving Mathematical Problems: A Personal Perspective*. Oxford University Press, Inc., New York.	This book was originally written by a 15 year old involved in math competitions. The material is probably better suited for high school or advanced middle school students. Covers strategies, number theory, algebra, geometry, and other material.
Engel, A. 1998. Problem-Solving Strategies. Springer, New York.	This book is targeted at preparation for the "highest level of international competitions, including the IMO *[International Mathematical Olympiad]* and the Putnam Competition."
Rubinstein, M.F., and I.R. Firstenberg. 1995. *Patterns of Problem Solving*. Prentice Hall, Englewood Cliffs, New Jersey.	This book gives a broader view of problem solving. Covers general problem solving, language/communication, human memory, probability, and models (including decision making models, optimization of models, and dynamic system models).

Videos

Videos can be useful not only for instruction, but to also provide some variety in meetings. You can find video resources at local libraries and online (including YouTube). A selection of video material is listed below. Several websites discuss where mathematics has been used or referred to in movies and television. Two series that provide instruction on math topics while making the information fun, interesting, and/or related to the real world include the Discovery Education videos and videos by the Standard Deviants. There are at least two PBS documentaries related to math that are interesting. And there are a number of shorter clips involving math in a humorous way (in addition to a wealth of tutorial type videos).

Website	URL
Math in Hollywood	
"Math and the Movies" Part One	mathbits.com/mathbits/mathmovies/ResourceList.htm
Mathematics in Movies	www.math.harvard.edu/~knill/mathmovies/index.html
Educational Series	
Discovery Education – Concepts in Advanced Algebra, Concepts in Number Theory, Concepts in Geometry	store.discoveryeducation.com/product/show/49008 store.discoveryeducation.com/product/show/50778 store.discoveryeducation.com/product/show/49161
Standard Deviants – Basic Math, Pre-Algebra, Algebra, Geometry	www.sdlearn.com/Math_s/7.htm
MATHCOUNTS Minis	mathcounts.org/Page.aspx?pid=1513
Documentaries	
The Story of 1	www.pbs.org/previews/storyof1
Fractals: Hunting the Hidden Dimension	www.pbs.org/wgbh/nova/fractals/program.html
Examples of Online Clips	
Arthur Benjamin's TED Talk	www.math.hmc.edu/~benjamin/mathemagics.htm
Ma & Pa Kettle Math Lesson	video.google.com/videoplay?docid=7106559846794044495
Abbot & Costello	www.youtube.com/watch?v=rLprXHbn19I
Simpsons	www.hulu.com/watch/29527/the-simpsons-aptitude-test-cheater
Audio	
BBC Radio 4 Mathematics Podcasts	www.bbc.co.uk/radio4/collections/mathematics